CYCLE TIME OPTIMIZATION FOR IMPERFECT MANUFACTURING SETUP FOCUSING WORK-IN-PROCESS INVENTORY

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ABSTRACT

Imperfect production processes have been focused during recent decades for optimum lot size calculation based on average cost minimization. However, cycle time optimization has been ignored relatively when processes are imperfect and inspection as a process is considered. Furthermore, the role of work-in-process inventory with respect to cycle time has always been significant. Hence, this paper integrates work-in-process inventory and imperfect production setup in order to optimize cycle time based on average cost minimization. A mathematical model is developed that incorporate rework operation, rejected products produced, and inspection processes in addition to work-in-process inventory. Cycle time is optimized based on total system cost minimization. Numerical example is also used to illustrate the impact of the developed model as compared to the previously developed model. The impact of work-in-process inventory and processes imperfection on optimum cycle time is highlighted by an example.

Keywords: *Cycle time, work-in-process inventory, rework, rejection, optimization.*

INTRODUCTION

Inventory models have been extensively used in calculation of optimum lot size based on cost minimization. Significant research has been carried out in the past several years, particularly in the field of inventory control as processes are affected by different factors. Basic inventory models primarily focus perfect manufacturing environments. Most of the models lack in their ability to address the scenarios when process are imperfect and produce defective products in the form of reworkable or rejected products. Hence, several extensions have been made to the basic models in order to calculate optimum lot size based on average cost minimization.

The economic order quantity (EOQ) model has been considered as the basic model among the lot size formulation since 1913[1]. This model calculates optimum lot size based on minimization of ordering and carrying costs with several assumptions. Few of these assumptions include that inventory is received instantaneously and no shortages are allowed. It was assumed that demand rate is known and remains constant. However, it was realized later that inventory is not received instantaneously rather buildup as production continue and demands are fulfilled with the passage of time in parallel. Hence, a production order quantity model (POQ) was developed for a production environment based inventory system in 1918[2]. This model assumed that the production rate is always greater than the demand rate. The excess amount of products produced during the production process develops inventory, which is carried during the production process. These models are considered as the backbone of inventory control and management and have been extensively used for lot size calculation even after hundred years.

Numerous extensions have been made to the above mentioned basic models as these models were based on some unrealistic assumptions. Models extensions were made to explain more practical scenarios in daily business life. One major assumption of these inventory models (EOQ and POQ) is that the process always produces perfect quality products. Variation in quality across batches of raw materials, material internal defects, tool wear and tear, machine failures and environmental conditions are the common factors that might affect the manufacturing processes. These factors make it difficult to deliver perfect quality products every time. Process goes out-of-control state and produce imperfect products. Imperfect products produced during the process are either in the form of reworkable or non-reworkable. Reworking of imperfect production units has been analyzed in details with different assumptions during last several years. Researchers put their efforts to develop models that can address industrial problems. Rosenblatt and Lee[3] are among those pioneers who considered imperfect production setup with economic production cycles. Their

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model assumed that the production units are deteriorated during the production process. Deterioration results in defective products. Cycle time was optimized and it was concluded that the developed model results in reduced cycle time as compared to the classical inventory model. Gupta and Chakraborty[4] calculated optimum batch size focusing multistage production system with emphasis on recycling process. It was assumed that recycling process is performed from the last stage to the first one. They calculated optimal batch size and recycle lot size based on operation cost.

Porteus[5] made a significant contribution towards imperfect production setup by highlighting the fact that smaller lot sizes would increase system wide benefits and improved quality control. A relationship between lot size and quality was developed as the production facility bears additional cost of rework when process goes into out-of-control state and produce poor quality products. They stressed on investment in quality improvement techniques to get fewer defective products during manufacturing processes. Boucher[6] introduced the concept of group-technology work environment and computed optimum lot size based on average cost minimization. They emphasized on the fact that in large discrete manufacturing environments, manufacturing take longer time and product worth increases as it moves from one station to another station due to value-addition. Cheng[7] further extended the work related to economic order quantity model by assuming that production cost is demand-dependent and processes are imperfect. Process reliability was considered to produce quality product. Closed form solution was obtained by considering this inventory problem as geometric program. Gunasekaran et. al.,[8] developed mathematical model for optimal lot size and investment to improve productivity and quality of a production setup. They employed smaller lot size and quality control approach to reduce cycle time and defective products production. Optimization was based on total system wise cost minimization.

Liu et. al.,[9] emphasized on the fact that rework operation, in some complex scenarios, cannot be performed on a separate machine. They developed a mathematical model for optimal lot size by considering an imperfect single stage manufacturing system. It was assumed that rework operation is performed at the end when whole lot production is completed. They further assumed that inspection is automatic and consume no time. Lin[10] developed an integrated production-inventory model assuming imperfect production processes with raw materials constraints. The model assumes that defective items are produced during the production process. They further assumed that defective items are function of setup cost. Near optimal solution was obtained based on total cost function. Ben-Daya and Hariga[11] focused the impact of imperfect production units production on scheduling problems. Mathematical model was developed for economic lot schedule by incorporating imperfect quality and process restoration. Many researchers[11-15,17-26] developed such kind of models for real world scenarios with imperfect production processes.

This research has focused on the cycle time optimization for an imperfect production setup focusing work-in-process inventory. Previous studies have mainly focused lot size calculation rather than the cycle time for an imperfect production setup. The importance of cycle time is also significant when work-in-process inventory is taken into consideration. Furthermore, this is substantial when manufacturing system is having imperfection i.e., process is producing reworkable products, products are rejected, and inspection processes are involved.

The remaining parts are organized as follow: Next section define the mathematical modeling with used notation and assumptions. Section 3 is devoted to the illustrations by a numerical example with results and discussions while section 4 presents the conclusions and directions for future research.

MATHEMATICAL MODELING

2.1 Notation

Following notation and assumptions are used in this mathematical model are as under:

Parameters

D customer demand in a cycle time (units/unit time).

Q lot size (units)

C_M raw material cost per unit product ($/unit).

C_p purchase cost per unit of cycle time ($/unit of time).

C_S setupcost per unit of cycle time ($/unit of time).

C_i inspectioncost per unit of cycle time ($/unit of time).

C_{wip} *work-in-process* holding cost per unit of cycle time ($/unit of time).

C_h inventory holding cost per unit of time ($/unit of time).

C_{total} total cost per unit of cycle time ($/unit of time).

S setup time for each lot (unit time/setup).

m_1 machining time for each product in regular production phase (time/unit product).

m_r machining time for each *re-workable* product (time/unit *re-workable*product).

P_{10} poor quality products produced in regular production phase (%).

P_{11} perfectquality products produced in regular production phase (%).

P_{21} perfect quality products produced in rework phase(%).

P_{20} poor quality products produced in rework phase (%).

P_{1r} *re-workable* products produced in cycle time (%).

P_b poor quality products in each cycle (%).

P_g Perfect quality products produced in each cycle (%).

T_p processing time.

T average manufacturing time for each product item.

I average inventory at the end of each cycle (units).

W average monetary value of the WIP inventory ($).

i inventory holding cost per unit of time ($/unit of time).

c average unit value of each product cost (unit of money ($/unit product).

R rate charged per unit of cell production time including all overheads, moving cost, loading/unloading cost etc. ($/unit of time).

Decision Variable

T_c Cycle time per unit lot size.

2.2 Assumptions

Mathematical modeling of the problem is based on following assumptions.

Imperfect items are produced during regular production phase. Some percentages of products are re-workable and some are rejected during regular phase of lot manufacturing.

Re-workable products are processed again in order to get qualified products from these re-workable products. However, rework process also results in some percentage of rejected products.

Inspection operation is performed at the end of regular production and at the end of rework operation. Both inspection processes consume time and resources.

It is assumed that value addition occur as it moves from raw-material stage to the final product shape either reworked, rejected, or qualified.

Production rate is always greater than the customer demands even the processes are imperfect. Customers get quality products only.

Demand is known and continuous.

Shortages are not allowed during the whole cycle time.

2.3 Mathematical Problem Formulation

The mechanism of a production setup for this problem is shown in Figure 1. Lot size Qreaches the manufacturing station and is processed for operation as defined as per process plan. Products produced at the station are sent for inspection purposes where features produced in the product are checked as per predefined specifications. Some percentage of products are reworkable where as some percentage is rejected. Reworkable products are processed again and result in some percentage as qualified.

This article optimize cycle time for an imperfect manufacturing setup focusing work-in-process inventories that has been ignored relatively in economic and production order quantity models.

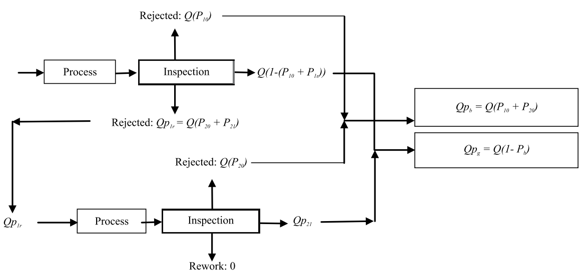

Figure 1. Production setup with processes imperfection

Cycle time comprises of the following sub-components.

t_1= Production process uptime in which production process take place on one side whereas raw-materials/ subassemblies are processed for their turn on machine, on the other side.

t_2= During this time, products produced during the regular production phase are passed through an inspection stage.

t_3= It's the time required for rework operation to be performed for re-workable products.

t_4= Time consumed in the inspection of re-workable products.

t_5= Downtime for manufacturing setup. However, customer demands are fulfilled during this stage.

We know that cycle time for group-technology manufacturing setup is given by the following relation,

$$T_c = t_1 + t_2 + t_3 + t_4 + t_5 \tag{1}$$

where

$$T_c = \frac{Q}{D} \tag{2}$$

$$t_1 = \frac{Q}{P} \tag{3}$$

$$t_2 = \frac{Q}{1r} \tag{4}$$

$$t_3 = \frac{Q(P_1 r)}{P} \tag{5}$$

$$t_4 = \frac{Q(P_1 r)}{1r} \tag{6}$$

Therefore,

$$t_5 = T_c - (t_1 + t_2 + t_3 + t_4)$$

$$t_5 = \frac{Q\left\{1-\left(\frac{D}{P}+\frac{D}{1r}+\frac{Dp1r}{P}+\frac{Dp1r}{1r}\right)\right\}}{D}$$

$$t_5 = \frac{Q\left\{1-\frac{D}{P}\left(1+P_{1r}\right)-\frac{D}{1r}\left(1+P_{1r}\right)\right\}}{D} \tag{7}$$

The inventories associated with this model comprise of work-in-process inventory and finish products inventory. Work-in-process inventory comprises raw material or subassemblies waiting for operation on machines which are consumed at the production rate during the production phase of the manufacturing cycle. Rejected products are also produced during the same cycle period when raw-materials were converted into finish goods due to process imperfection. Simultaneously, inventory of finish products within the manufacturing setup also developed. Therefore, the average work-in-process inventory carried during the production phase is given by

$$\bar{W} = \frac{QD}{2P}\left(C_M\right) + \frac{Q(1-P_b)\frac{D}{P}}{2}\left(C\right) + \frac{Q(P_b)\frac{D}{P}}{2}\left(C\right)$$

$$\bar{W} = \frac{D^2}{2P}\left(C_M + c\right) \tag{8}$$

where 'c' is the average value added to the product as it moves from one production station to another. It is calculated as under[25]:

$$c = C_M + R\left(\frac{s + Qm_1 + 1Q + Qm_r p_{1r} + 1Qp_{1r}}{Q}\right)$$

We know that the cost associated with work-in-process inventory is given by [Silver et. al., 1998]:

$$C_{wip} = \bar{W}$$

Therefore,

$$C_{wip} = \frac{D^2 i}{2P}\left(C_M + c\right)\left(T_c\right) \quad (9)$$

Similarly, cost of carrying cost per unit cycle time during the production down period is obtained using following relation[26]:

$$Ch = ic\bar{I}$$

Where

$$\bar{I} = \frac{\left(\frac{Q(1-P_b)}{2}\right)\left(\frac{Q\{1-\frac{D}{P}(1+P_{1r})-\frac{D}{P}(1+P_{1r})\}}{D}\right)}{T_c}$$

$$\bar{I} = \left(\frac{DTc\ (1-P_b)}{2}\right)\left(1-\frac{D}{P}(1+P_{1r})-\frac{D}{P}(1+P_{1r})\right)$$

Hence,

$$Ch = ic\left(\frac{D\ (1-P_b)}{2}\right)\left(1-\frac{D}{P}(1+P_{1r})-\frac{D}{Ir}(1+P_{1r})\right)(T_c) \quad (10)$$

Other costs taken into consideration are the purchase cost, setup cost, and the inspection cost. Inspection cost for each product related to the time need for inspection. It is assumed that inspection cot per unit time is taken as unity.

$$C_p = \frac{A}{T_c} \quad (11)$$

$$C_i = ID\ (1 + P_{1r}) \quad (12)$$

And

$$C_i = ID\ (1 + P_{1r}) \quad (13)$$

Therefore, the total cost function per unit cycle time is given by

$$C_{total}\ (T_c) = C_M D + \frac{A}{T_c} + \frac{D^2 i}{2P}(C_M + c)\ (T_c) + ID(1 + P_{1r}) + ic\left(\frac{D\ (1-P_b)}{2}\right)\left(1-\frac{D}{P}(1+P_{1r})-\frac{D}{1r}(1+P_{1r})\right)(T_c) \quad (14)$$

Factorizing above equation

$$C_{total}\ (T_c) = \frac{A}{T_c} + \left\{\frac{D^2 i}{2P}(C_M + c) + ic\left(\frac{D(1-P_b)}{2}\right)\left(1-\frac{D}{P}(1+P_{1r}) - \frac{D}{P}(1+P_{1r})\right)\right\}(T_c) + C_M D + ID(1 + P_{1r}) \quad (15)$$

Equation (15) is of the form, y(x) + a1 (x) + a2/x + α3. Therefore, algebraic optimization method can be used to obtain the optimum cycle time $(T_c{*})$ by minimization of the total cost function (15).

$$x = \sqrt{\frac{a_2}{a_1}}$$

$$T_c{*} = \sqrt{\frac{2A}{\left[icD(1-P_b)\left(1-\frac{D}{P}(1+P_{1r})-\frac{D}{Ir}(1+P_{1r})\right)+\frac{D^2 i}{Ir}(C_M+c)\right]}} \quad (16)$$

Equation 16 gives the optimum cycle time for *WIP* based inventory model considering imperfection in processes with inspection. The global minimum cost function is given by

$$C_{total} = (T_c{*}) = 2\sqrt{a_1 a_2} + a_3$$

$$C_{total}(T_c{*}) = 2\sqrt{\left(icD(1-P_b)\left(1-\frac{D}{p}(1+P_{1r})-\frac{D}{Ir}(1+P_{1r})\right)+\frac{D^2 i}{P}(C_M+c)\right)(2A)} + (C_M D + ID(1+P_{1r})) \quad (17)$$

Special case: Optimum cycle time obtained by using equation (16) can be reduced to the cycle timeof classical economic production lot size model having perfect production processes without considering work-in-process inventory. Furthermore, when inspection rate is very high then time for inspection is very low. The optimum cycle time for production order quantity model is given by

$$T_c{*} = \sqrt{\frac{2A}{icD(1-\frac{D}{P})}} \quad (18)$$

3. ILLUSTRATION WITH NUMERICAL EXAMPLE

For illustration of the proposed model for the cycle time optimization based on average minimum cost, following example has been used. It is assumed that production rate and inspection rate are higher than the demand rate.

Example. Different parameters taken into consideration with appropriate units are as under: demand rate (D) = 12000 (units per year), inspection rate is taken as 40000 units per year, value added during production process is taken as (c) = 2.5 (\$/unit), Inspection cost (Ic) = 1 (\$/unit), setup cost (A) = 8.9 (\$/unit), setup time (s) = 0.0012 (years/unit), charges per unit of cell production (R) = 2000 (\$/unit), material cost (C_M) = 1 (\$/unit), i= 30%. It is assumed that the expected rejection rate (P_b) during the lot manufacturing ranges from 0% to 50% and the expected reworkable products produced (P_{1r}) are assumed to vary from 0% to 25%.

The average cycle time has been shown for three different conditions. Cycle time (T_{c1}) represents the cycle time as calculated by the analytical solution obtained in equation (16). It has incorporated imperfection and work-in-process inventory in calculation. The next cycle time (T_{c2}) is the cycle time calculated under the ideal condition where no imperfection exists in process as calculated by Equation (18). Furthermore, it does not incorporate work-in-process inventory. The third cycle time (T_{c3}) is calculated when work-in-process inventory is not taken into consideration. Only the impact of imperfection is incorporated into the ideal conditions. Cycle time increases with increase in imperfection in the process in the form of rejection and rework.

It can be observed that the impact of work-in-process inventory on cycle time has been highly significant. Only considering imperfection in process significantly increases the average cycle time as shown by cycle time-T_{c3}. Cycle time T_{c2} has not been changed as it is not affected by the process imperfection.

Table 1 also highlight the impact of imperfection on total cost function. Average total cost increases with increase in imperfection rate.

Table 1. Average cycle time and expected cost calculation with processes imperfection.

S N	Pb	P1r	Cycle time (Tc1) (Hrs)	Perfect Cycle time (Tc2)(Hrs)	Cycle time Without WIP (Tc3) (Hrs)	Total cost ($)
1	0	0	95.91	114.83	162.39	24371.17
2	10	5	99.54	114.83	182.13	24957.65
3	20	10	103.12	114.83	207.35	25545.23
4	30	15	106.58	114.83	240.74	26134.03
5	40	20	109.82	114.83	287.07	26724.17
6	50	25	112.73	114.83	355.78	27315.79

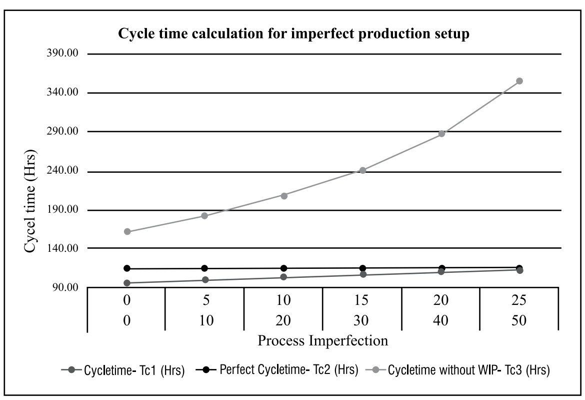

Figure 2. Variation of cycle time with variation in imperfection

Figure 2 highlights all three types of cycle times calculate under different scenarios as observed in Table 1. It can be observed that by taking work-in-process inventory into consideration, the impact of imperfect process on average cycle time has been reduced significantly. Based on the developed model, cycle time is reduced as comapred to the ideal ecomic order quantity model. Most of the literature assume imperfection in process while calcualting average cycle time. However, they neglect the significant effect of work-in-process inventory on average cycle time calcualtion. This example provide a detailed explananation regarding the imporatnce of work-in-process into consideration when developing such type of models.

4. CONCLUSIONS

This paper highlighs the significant impact of work-in-process inventory on cycle time calcualtion. A mathematical model was developed for an imperfect production setup where value-added process were-considered as product moves from raw material stage towards final product. Processes were assumed to be imperfect and inspection process was assumed to ensure that customer receive only quality products. Cycle time was optimized based on average cost minimization. Numerical examples were used to highlight the impact of work-in-proces inventory on cycel time calculation under imperfect production setup. It was observed that the cycle time and total expected cost has been increased as processes deviates from the ideal condiions. Cycle time was reduced as comapred to the conventional inventory model. Therefore, the article provides insight to practitioners by integrating process imperfection with inspection processes and work-in-process inventory. The paper can be further extended by incorporating shortages and preventive maintanance.

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